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Isospin Analysis of Two-body B Decays and Test of Factorization

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Abstract

It is shown that the existing data on two-body B decays, some of them only upper limits, are precise enough to perform an isospin analysis to extract the phase shifts due to final state interaction. Unlike charm decays, no significant final state interaction is observed in decays $B \to D\pi, D\rho$, and $D^*\pi$ supporting the factorization hypothesis in these decays. From the isospin amplitudes obtained, we extract the ratio a_2/a_1, where a_1 and a_2 are the coefficients in the factorized effective Hamiltonian.

The idea of factorization has been used for evaluating non-leptonic weak decays ever since Schwinger employed it to show that the $\Delta I = 3/2$ kaon decay rate is consistent with the corresponding semileptonic decay rate [1, 2]. Even though the method was originally thought to be useful only for order of magnitude estimations, it has been extensively applied to heavy hadron decays [3, 4, 5, 6] with results varying from mixed to reasonable. Since factorization is thus far virtually the only way to quantitatively calculate exclusive non-leptonic rates, it is important to test the validity of the hypothesis whenever possible; in fact, it has been a subject of a number of studies [7].

We take the decay $\overline{B}^0 \to D^+\pi^-$ as an example which can occur by the 4-fermion operator [5]

$$a_1(\overline{d}u)_\mu(\overline{c}b)^\mu + a_2(\overline{c}u)_\mu(\overline{d}b)^\mu \tag{1}$$

where the notation is $(\overline{q}q')_\mu \equiv \overline{q}\gamma_\mu(1-\gamma_5)q'$ and $a_{1,2}$ are real coefficients which are related to the Wilson coefficients C_1 and C_2 by

$$a_1 = C_1 + \xi C_2 \qquad a_2 = C_2 + \xi C_1. \tag{2}$$

The parameter ξ is sometimes called the color suppression factor and naively expected to be 1/3. At the mass scale of b-quark, the leading-logarithm approximation gives [8]

$$C_1(m_b) = 1.11 \qquad C_2(m_b) = -0.26. \tag{3}$$

The basic idea of factorization is that the pion is generated from vacuum by the current operator $(\overline{d}u)_\mu$ and the transition $B \to D$ is caused by the current operator $(\overline{c}b)^\mu$, and that they occur independently. In terms of matrix element, it amounts to the fact that it can be written in a factorized form:

$$Amp(\overline{B}^0 \to D^+\pi^-) = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{cb}\, a_1 \langle \pi^- | (\overline{d}u)^\mu | 0 \rangle \langle D^+ | (\overline{c}b)_\mu | \overline{B}^0 \rangle. \tag{4}$$

Such assumption of factorization has been shown to be correct to the zeroth order in $1/N$ expansion [9]. Also, it has been argued intuitively that factorization should hold for energetic tow-body decays [3, 12] based on two observations: 1) when the $u\overline{d}$ pair escapes the color field around the b quark, it is highly energetic (~ 2.5 GeV) and, since it has to eventually form a pion, the pair is collinear and close together with the total color being zero. Thus, we expect that the pair will escape the color field without much interaction ('color transparency'). 2) By the time the pion is formed, it will be well outside the color field (again due to the high energy of the pion); thus, there will be little final state interaction (FSI) between the D meson and the pion. The same arguments had been used for ρ and ψ productions in hard scatterings [13], and recently put forward further by Dugan and Grinstein in the framework of heavy quark effective theory [14].

Since factorization assumes independence of the pion formation and the B to D transition, FSI between them is antithesis to factorization. Often, however, factorization and FSI are combined to be compared with data assuming that the factorization calculation correctly estimates the amplitude 'just before' FSI takes place [6, 15]. This procedure has worked reasonably well for charm decays, and we will effectively employ it later when we extract a_2/a_1. It is worth keeping in mind, however, that such treatment is not justified in the $1/N$ expansion [9] and also it is not well defined exactly where factorization ends and FSI begins.

In the isospin analysis of the charm decays $D \to K\pi$ [6], it is found that the FSI phase shift between isospin 1/2 amplitude and isospin 3/2 amplitude is large ($\sim 77°$). Furthermore, after the effect of FSI is removed as described above, the coefficients a_1 and a_2 are found to be nearly identical to the values of Wilson coefficients C_1 and C_2 evaluated at the charm mass scale, corresponding to $\xi \sim 0$. This has led to the rule of so-called 'discarding 1/N terms' [9] and prompted further theoretical studies based on QCD sum rules [11]. A recent analysis [10], however, indicates that the situation is quite different for B decays giving the value of ξ around 1/2 to 1/3. In this study, we will see that the existing data leads us to conclude that the FSI phase shifts are small for the B decays $B \to D\pi, D\rho$, and $D^*\pi$, and that the value of ξ remains to be 1/2$\sim$1/3 even after the effect of FSI is taken out.

The Hamiltonian responsible for $B \to D\pi$ decays has isospin $|1,-1\rangle$, and this leads to the following isospin relations:

$$
\begin{aligned}
A^{+-} &= \sqrt{\frac{1}{3}}A_{\frac{3}{2}} + \sqrt{\frac{2}{3}}A_{\frac{1}{2}} \\
A^{00} &= \sqrt{\frac{2}{3}}A_{\frac{3}{2}} - \sqrt{\frac{1}{3}}A_{\frac{1}{2}} \\
A^{0-} &= \sqrt{3}A_{\frac{3}{2}}
\end{aligned}
\tag{5}
$$

where $A^{+-} \equiv Amp(\overline{B}^0 \to D^+\pi^-)$, $A^{00} \equiv Amp(\overline{B}^0 \to D^0\pi^0)$, $A^{0-} \equiv Amp(B^- \to D^0\pi^-)$, and $A_{3/2}$, $A_{1/2}$ are the isospin 3/2 and 1/2 amplitudes, respectively. There are three unknown parameters: $|A_{3/2}|$, $|A_{1/2}|$, and $\delta = \arg(A_{3/2}/A_{1/2})$. Since there are three measurements of decay rates (namely, $|A_{+-}|^2$, $|A_{00}|^2$, and $|A_{0-}|^2$), one can solve for the three unknowns:

$$
\begin{aligned}
|A_{\frac{3}{2}}| &= \sqrt{\frac{|A_{0-}|^2}{3}} \\
|A_{\frac{1}{2}}| &= \sqrt{|A_{+-}|^2 + |A_{00}|^2 - \frac{|A_{0-}|^2}{3}} \\
\cos\delta &= \frac{|A_{+-}|^2 + |A_{00}|^2 - |A_{0-}|^2/3}{\sqrt{\frac{8}{3}|A_{0-}|^2(|A_{+-}|^2 + |A_{00}|^2 - |A_{0-}|^2/3)}}
\end{aligned}
\tag{6}
$$

Note also that the isospin relations (5) can be expressed as a single triangle relation:

$$
A^{+-} + \sqrt{2}A^{00} = A^{0-}. \tag{7}
$$

The same relations hold for the decays $B \to D^*\pi$ and $B \to D\rho$. For the decay $B \to D^*\rho$ the same relations hold separately for each helicity amplitude. Since there is no a priori reason to believe that the polarization is the same for $D^{*+}\rho^-$, $D^{*0}\rho^0$, and $D^{*0}\rho^+$, and since there is not enough data to separate the helicity amplitudes, we will not include $D^*\rho$ mode in this analysis.

Amplitudes calculated by factorization naturally satisfy the triangle isospin relation (7). This can be seen from the expression (4) and corresponding factorized forms for A_{+-} and A_{00}:

$$
\begin{aligned}
A^{00} &= \frac{G_F}{\sqrt{2}}V_{ud}^*V_{cb}a_2\langle D^0|(\overline{c}u)^\mu|0\rangle\langle\pi^0|(\overline{d}b)_\mu|\overline{B}^0\rangle \\
A^{0-} &= \frac{G_F}{\sqrt{2}}V_{ud}^*V_{cb}\Big[a_1\langle\pi^-|(\overline{u}d)^\mu|0\rangle\langle D^0|(\overline{c}b)_\mu|\overline{B}^-\rangle \\
&\qquad +a_2\langle D^0|(\overline{c}u)^\mu|0\rangle\langle\pi^-|(\overline{d}b)_\mu|\overline{B}^-\rangle\Big]
\end{aligned}
$$

and noting that (from isospin symmetry)

$$
\begin{aligned}
\langle D^0|(\overline{c}b)_\mu|\overline{B}^-\rangle &= \langle D^+|(\overline{c}b)_\mu|\overline{B}^0\rangle \\
\sqrt{2}\langle\pi^0|(\overline{d}b)_\mu|\overline{B}^0\rangle &= \langle\pi^-|(\overline{d}b)_\mu|\overline{B}^-\rangle.
\end{aligned}
$$

In fact, the isospin amplitudes are explicitly given by

$$A_{\frac{3}{2}} = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{cb} \frac{1}{\sqrt{3}} (M_1 + M_2) \tag{8}$$

$$A_{\frac{1}{2}} = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{cb} \sqrt{\frac{3}{2}} \Big(\frac{2}{3} M_1 - \frac{1}{3} M_2 \Big)$$

or

$$\frac{A_{3/2}}{A_{1/2}} = \sqrt{2} \frac{1 + M_2/M_1}{2 - M_2/M_1} \tag{9}$$

where

$$M_1 \equiv a_1 \langle \pi^- | (\overline{d}u)_\mu | 0 \rangle \langle D^+ | (\overline{c}b)^\mu | \overline{B}^0 \rangle \tag{10}$$

$$M_2 \equiv a_2 \langle D^0 | (\overline{c}u)_\mu | 0 \rangle \langle \pi^- | (\overline{d}b)^\mu | B^- \rangle$$

As before, the same relations (8-10) hold for decays $D^*\pi$, $D\rho$, and each helicity state of $D^*\rho$.

If factorization is assumed, the three amplitudes A_{+-}, A_{00}, and A_{0-} are relatively real, and thus the triangle (7) reduces to a line. Therefore, if the measured decay rates are exactly as expected from factorization as prescribed above, then the isospin analysis is guaranteed to give $\delta = 0$. Actual measurements, however, are always associated with errors, and the main point of this article is in showing that meaningful isospin analyses can be performed even though only upper limits are available for some of the decay modes.

The isospin amplitudes cannot be uniquely given by factorization since it depends on decay constants and form factors through (10). If we use the model of Bauer, Stech and Wirbel [6, 17] together with $f_\pi = 132$ MeV and $f_D = 220$ MeV, we obtain

$$\frac{M_2}{M_1} = \begin{cases} 1.23\, a_2/a_1 & (D\pi) \\ 1.30\, a_2/a_1 & (D^*\pi) \\ 0.66\, a_2/a_1 & (D\rho) \end{cases} \tag{11}$$

where the a_1, a_2 are the coefficients appearing in the effective Hamiltonian (1). This can be substituted in (9) to obtain the expected ratio of isospin amplitudes, or if the ratio $A_{3/2}/A_{1/2}$ is known, a_2/a_1 can be extracted from

$$\frac{M_2}{M_1} = 2 \frac{A_{3/2}/A_{1/2} - 1/\sqrt{2}}{A_{3/2}/A_{1/2} + \sqrt{2}}. \tag{12}$$

The table 1. shows the current available measurements for the relevant decay modes [10]. We will take the statistical errors only, and assume that the life times of $\overline{B}^0$ and B^- are the same:

$$\tau(\overline{B}^0) = \tau(B^-) = 1.18\text{ps}. \tag{13}$$

The upper limits are converted to a gaussian distribution centered at zero by setting the r.m.s. of the gaussian to (upper limit)/1.64.

$\overline{B}^0$ mode	(%)	$\overline{B}^0$ mode	(%)	B^- mode	(%)
$D^+\pi^-$	0.29 ± 0.04	$D^0\pi^0$	< 0.035	$D^0\pi^-$	0.55 ± 0.04
	$\pm 0.03 \pm 0.05$				$\pm 0.03 \pm 0.02$
$D^{*+}\pi^-$	0.26 ± 0.03	$D^{*0}\pi^0$	< 0.072	$D^{*0}\pi^-$	0.49 ± 0.07
	$\pm 0.03 \pm 0.01$				$\pm 0.06 \pm 0.03$
$D^+\rho^-$	0.81 ± 0.11	$D^0\rho^0$	< 0.042	$D^0\rho^-$	1.35 ± 0.12
	$\pm 0.12 \pm 0.13$				$\pm 0.12 \pm 0.04$

Table 1: Branching ratios measured by CLEO. The first error is statistical, the second error is systematic, and the third error is due to uncertainties in D branching ratios. The upper limits are 90% confidence levels.

Table 2 shows the solution for $|A_{3/2}|, |A_{1/2}|$ and $\cos\delta$ using the formulae (6). For each mode, $\cos\delta$ is consistent with unity indicating that there is no phase shifts due to final state interaction. In this analytical method, however, the range of $\cos\delta$ is not constrained to within ± 1. In order to take the constraint into account properly, we will use the maximum likelihood method. The likelihood function for $|A_{3/2}|$, $|A_{1/2}|$, and $\cos\delta$ is given by

$$L = N \prod_{i=+-,00,0-} \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left(\frac{(\Gamma_i - \Gamma_i^0)^2}{2\sigma_i^2}\right) \tag{14}$$

where Γ_i and σ_i are the measured decay rate and its error, Γ_i^0 is the decay rate calculated from the amplitude A^i given by (5) according to the standard formula

$$\Gamma_i^0 = \frac{p}{8\pi M_B^2}|A^i|^2, \tag{15}$$

and the normalization factor N is added to make the integral over the allowed region of $|A_{3/2}|$, $|A_{1/2}|$, and $\cos\delta$ to be unity. Result of the fit is also shown in Table 2. Because of the constraint on $\cos\delta$, the errors are generally better than those of the analytical solutions. For each mode, the most likely value for $\cos\delta$ was unity. Figure 1 show the 1,2 and 3 sigma contours for $|A_{3/2}|$ vs $|A_{1/2}|$ and $|A_{3/2}|$ vs $\cos\delta$. It is seen that the parameters are not strongly correlated.

Also shown in Table 2 are the ratio of isospin amplitudes $|A_{3/2}/A_{1/2}|$ and a_2/a_1 extracted using equations (11-12). For each mode, the ratio a_2/a_1 is positive which is a consequence of $|A_{1/2}| < \sqrt{2}|A_{3/2}|$. Averaging over the three modes, we obtain

$$\frac{a_2}{a_1} = +0.25 \pm 0.05 \tag{16}$$

or from (2) the corresponding color suppression factor ξ is

$$\xi = 0.45 \pm 0.04 \tag{17}$$

Figure 1: One, two and three sigma contours for the 2-dimentional plots of the likelihood function: $|A_{3/2}|$ vs $|A_{1/2}|$ (a) and $|A_{3/2}|$ vs $\cos\delta$ (b).

		$D\pi$	$D^*\pi$	$D\rho$
analytical	$\lvert A_{3/2}\rvert$ (10^{-5} GeV)	0.556 ± 0.021	0.533 ± 0.038	0.886 ± 0.040
solution	$\lvert A_{1/2}\rvert$ (10^{-5} GeV)	0.425 ± 0.095	0.411 ± 0.127	0.792 ± 0.133
	$\cos\delta$	1.20 ± 0.28	1.19 ± 0.61	1.11 ± 0.13
maximum	$\lvert A_{3/2}\rvert$ (10^{-5} GeV)	0.550 ± 0.020	0.527 ± 0.037	0.862 ± 0.037
likelihood	$\lvert A_{1/2}\rvert$ (10^{-5} GeV)	0.503 ± 0.057	0.494 ± 0.065	0.907 ± 0.084
	$\cos\delta^*$	> 0.82	> 0.57	> 0.92
$\lvert A_{3/2}/A_{1/2}\rvert$		1.09 ± 0.13	1.07 ± 0.16	0.95 ± 0.10
a_2/a_1		0.25 ± 0.07	0.22 ± 0.08	0.31 ± 0.11

* In all cases the most likely value for $\cos\delta$ is unity. The lower limits are at 90% confidence level.

Table 2: Analytical solutions and results of the maximum likelihood fit for the isospin amplitudes and their relative phase angle. Also given are the ratio of the isospin amplitudes and a_2/a_1 derived therefrom (using the result of the maximum likelihood fit).

which is consistent with the analysis of Ref [10] where the decay rates were fit to the model by Bauer, Stech and Wirbel without taking out the final state interaction. This, however, cannot be considered to be an independent confirmation of the positive value of a_2/a_1 since the two analyses are highly correlated.

In summary, we have performed an isospin analysis on two-body B decays and found that the phase shifts by final state interaction are small in stark contrast to the case of charm decays. By fitting to the obtained isospin amplitudes, we have also seen that the effect of removing the final state interaction does not alter the observation that the ratio a_2/a_1 is positive.

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References

[1] J. Schwinger, Phys. Rev. Lett. **12** (1964) 630.

[2] R. P. Feynman, 'Symmetries in Elementary Particle Physics' Ed. A. Zichichi, p111 (Academic Press, 1965).

[3] D. Fakirov and B. Stech, Nucl. Phys. **B133** (1978) 313.

[4] A. Ali, J. Korner, G. Kramer, and J. Willroft, Z. Phys. **C1** (1979) 269.

[5] M. Bauer and B. Stech, Phys. Lett. **B152** (1985) 380.

[6] M. Bauer, B. Stech, and M. Wirbel, Z. Phys. **C34** (1987) 103.

[7] D. Bortoletto, S. Stone, Phys. Rev. Lett. **65** (1990) 295; T. Mannel, W. Roberts, Z. Ryzak, **D 44** (1991) R18; Phys. Lett. **B 259** (1991) 359; X.-Y. Pham, X.-C. Vu, Phys. Rev. **D 46** (1992) 261; Z. Hioki, T. Hasuike, T. Hattori, T. Hayashi, Phys. Lett. **B299** (1993) 115; A. Deandrea, N. Di Bartolomeo, R. Gatto, Phys. Lett. **B318** (1993) 549; C. Reader and N. Isgur, Phys. Rev. **D47** (1993) 1007.

[8] M. K. Gaillard and B. W. Lee, Phys. Rev. Lett. **33** (1974) 108; G. Altarelli and L. Maiani, Phys. Lett. **32B** (1974) 351.

[9] A. Buras, J.-M. Gérard, and R. Rückl, Nucl. Phys. **B268** (1986) 16.

[10] CLEO collaboration, CLEO DRAFT 93-87C, submitted to Phys. Rev. **D**.

[11] B. Blok and M. Shifman, Nucl. Phys. **B389** (1993) 534; M. Shifman, Nucl. Phys. **388** (1992) 346.

[12] J. Bjorken, Nucl. Phys. **B** (Proc. Suppl.) **11** (1989) 325.

[13] S. Brodsky and A. Mueller, Phys. Lett. **B206** (1988) 685.

[14] M. Dugan and B. Grinstein, Phys. Lett. **B255** (1991) 583.

[15] F. Buccella, M. Forte, G. Miele, G. Ricciardi, Z. Phys. **C48** (1990) 47.

[16] J.G. Korner and G.R. Goldstein, Phys. Lett. **89B** (1979) 105.

[17] M. Neubert, V. Rieckert, B. Stech, Q. P. Xu, in 'Heavy Flavors' p286, ed. A. J. Buras and M. Lindner, World Scientific, 1992.

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